Leader Notes: Distance to the Fire Station

Purpose:

The participants will model the absolute value of a number through The Fire Station Problem and an absolute value function through the use of a CBR in The Relay Race. They will explore the effects of parameter changes on the absolute value parent function. They will make connections among the characteristics of an absolute value function and solutions to absolute value equations and inequalities.

Descriptor:

This phase has four parts. In the first part, participants will graph the distances from a fire station to other buildings on the same street to develop the concept of absolute value. In the second part, participants will explore absolute value functions concretely using the Calculator-Based Ranger to model the graph and properties of an absolute value function and numerically by examining the number patterns found in tables of data values. In the third part, participants will examine the effects of parameter changes on the vertex form of absolute value functions. In the fourth part, participants will make connections among the algebraic processes of solving absolute value equations and the graphs and functions related to those equations.

Duration:

2.5 hours

TEKS:

a5 **Tools for algebraic thinking**. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, pictorial, numerical, symbolic, graphical, and verbal), tools, and technology (including, but not limited to, calculators with graphing capabilities, data collection devices, and computers) to model mathematical situations to solve meaningful problems.

a6 **Underlying mathematical processes.** Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, language and communication, and reasoning (justification and proof) to make connections within and outside mathematics. Students also use multiple representations, technology, applications and modeling, and numerical fluency in problem-solving contexts.

2A.1 **Foundations for functions.** The student uses properties and attributes of functions and applies functions to problem situations.

2A.1A The student is expected to identify the mathematical domains and ranges of functions and determine reasonable domain and range values for continuous and discrete situations.

2A.1B The student is expected to collect and organize data, make and interpret scatterplots, fit the graph of a function to the data, interpret the results, and proceed to model, predict, and make decisions and critical judgments.

2A.2 **Foundations for functions.** The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations.

2A.2A The student is expected to use tools including factoring and properties of exponents to simplify expressions and to transform and solve equations.

2A.3 **Foundations for functions.** The student formulates systems of equations and inequalities from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the solutions.

2A.3A The student is expected to analyze situations and formulate systems of equations in two or more unknowns or inequalities in two unknowns to solve problems.

2A.3B The student is expected to use algebraic methods, graphs, tables, or matrices to solve systems of equations or inequalities.

2A.4 **Algebra and geometry.** The student connects algebraic and geometric representations of functions.

2A.4A The student is expected to identify and sketch graphs of parent functions, including linear ($f(x) = x$), quadratic ($f(x) = x^2$), exponential ($f(x) = a^x$), and logarithmic ($f(x) = \log_a x$) functions, absolute value of x ($f(x) = |x|$), square root of x ($f(x) = \sqrt{x}$), and reciprocal of x $\left(f(x) = \frac{1}{x}\right)$.

2A.4B The student is expected to extend parent functions with parameters such as a in $f(x) = \frac{a}{x}$ and describe the effects of the parameter changes on the graph of parent functions.

TAKS™ Objectives Supported:

While the Algebra II TEKS are not tested on TAKS, the concepts addressed in this lesson reinforce the understanding of the following objectives.

- Objective 1: Functional Relationships
- Objective 2: Properties and Attributes of Functions
- Objective 3: Linear Functions
- Objective 4: Linear Equations and Inequalities
- Objective 10: Mathematical Processes and Mathematical Tools

Materials:

Prepare in Advance: Copies of participant pages, copies of **The Fire Station Problem Graphic Sheets** taped together

Presenter Materials: Overhead graphing calculator, CBR

Per group: **The Fire Station Problem** G**raphic Sheets** taped together to represent a street, CBR, link cable, chart paper

Per participant: Copy of participant pages, graphing calculator

Explore

<u>Part 1:</u> The Fire Station Problem

Leader Notes:
In this part of the professional development, participants investigate the concept of the absolute value of a number by locating a building a specific distance from the fire station. The fire station graphic sheets must be taped together so that the fire station is in the center of the street and the car is on the left. Explain to participants that the graphic on their tables represent the main street in a city. Ask participants to read the opening paragraph of the fire station Problem and answer the questions that follow on their participant pages. At the end of Part 1, participants will be able to connect a context for the absolute value of a number to a graphical representation using linear functions and compare linear equations to the graph of an absolute value situation.

> **A fire station is located on Main Street and has buildings at every block to the right and to the left. You will investigate the relationship between the address number on a building and its distance in feet from the fire station. On average, a mile in the city is composed of 16 city blocks. So each city block is about 330 feet long (5280 feet ÷ 16 = 330 feet). Each building is centered on the block.**

1. Complete the table below that relates the address of a building (x) with its distance in feet from the fire station (y).

Address Number (x)	Distance in Feet from the Fire Station (y)
800	*1320*
900	*990*
1000	*660*
1100	*330*
1200	*0*
1300	*330*
1400	*660*
1500	*990*
1600	*1320*

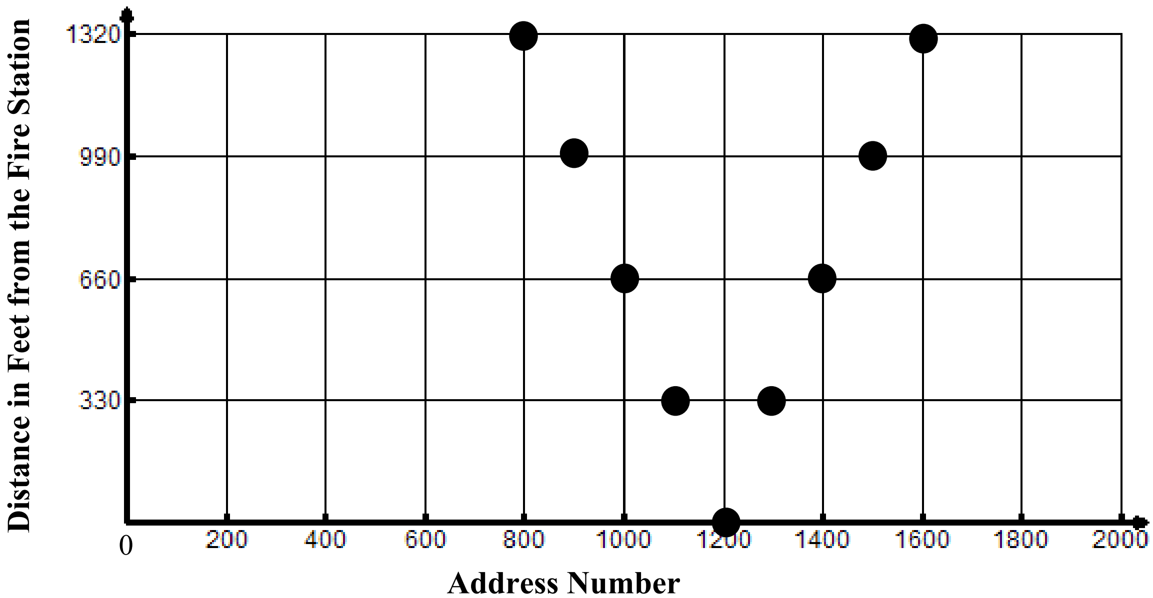
2. Which building is 660 feet way from the fire station? Explain your answer.
There are two buildings that are 660 feet from the fire station: the church and the ice cream shop. The church is 660 feet to the left of the fire station (as you look towards the fire station), and the ice cream shop is 660 feet to the right of the fire station.

3. If we send someone to the building that is 660 feet away from the fire station, how will she know that she has arrived at the correct place?
She will not know she has arrived at the correct place unless we tell her which direction to walk, or we indicate whether the building is to the right or the left of the fire station.

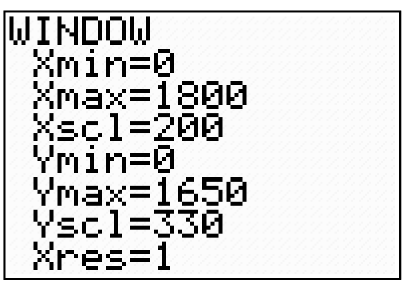
4. What words might we use to describe two locations that are the same distance from the fire station?
We describe them in terms of direction (north, south, east, west, 12 o'clock, 1 o'clock, etc.).

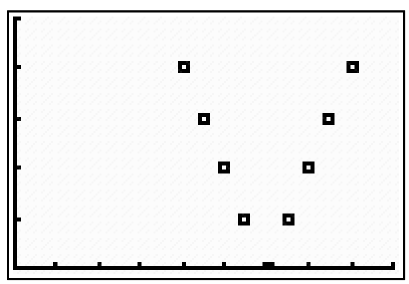
5. Suppose the buildings on Main Street are renumbered as if they are on a number line so that the location of the fire station represents 0. How do we describe two numbers with the same distance from 0?
We say that the two numbers have the same absolute value.

6. Draw a scatterplot that represents the data in the table.



7. Make a scatterplot of your data using your graphing calculator. Describe your viewing window.
Responses may vary. Possible responses are shown below.





8. What function or functions might the students use to describe the scatterplot?
Responses may vary. Students may use linear functions to describe the data. Students may also try to use a quadratic function to describe the data.

Leader note:
Algebra II teachers may assume that students have had prior instruction in absolute value. However, the concept of absolute value does not explicitly appear in the TEKS until Algebra II.

9. Find two linear functions that pass through the data points. What process did you use to find the equations of the lines?
The linear functions are $y = -3.3x + 3960$ and $y = 3.3x - 3960$. Possible processes may include table-building to find the y-intercepts, use of the slope formula to find the slope, use of point-slope, and use slope-intercept form. Possible approaches are described in detail on the next three pages.

First, we can find the equation of the line that passes through (800, 1320) and (1200, 0).

From the data table we can choose two points between (800, 1320) and (1200, 0), such as (900, 990) and (1000, 660), and use the formula for slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{660 - 990}{1000 - 900}$$

$$m = \frac{-330}{100}$$

$$m = -3.3$$

To find the y-intercept, we have three options: work backward in the table from (1200, 0) to the point (0, b) on the positive y-axis, use point-slope, or substitute a point and the slope into y = mx + b.

Option 1: Work backward in the table from (1200, 0) to the point (0, b) on the positive y-axis.

x	*y*
0	3960
100	3630
200	3300
300	2970
400	2640
500	2310
600	1980
700	1650
800	1320
900	990
1000	660
1100	330
1200	0

The y-intercept is 3960. Therefore, one equation is y = –3.3x + 3960.

Option 2: Using point-slope to find the y-intercept. We can choose the point (900, 990).

$$y - y_1 = m(x - x_1)$$
$$y - 990 = -3.3(x - 900)$$
$$y - 990 = -3.3x + 2970$$
$$y = -3.3x + 3960$$

Option 3: Substituting a point, such as (900, 990), and the slope into slope-intercept form and solving for b.

$$y = mx + b$$
$$990 = -3.3(900) + b$$
$$990 = -2970 + b$$
$$3960 = b$$
$$y = -3.3x + 3960$$

Now, we can find the equation of the line that passes through the points (1200, 0) and (1600, 1320).

From the data table we can choose two points between (1200, 0) and (1600, 1320), such as (1300, 330) and (1400, 660), and use the formula for slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{660 - 330}{1400 - 1300}$$
$$m = \frac{330}{100}$$
$$m = 3.3$$

To find the y-intercept, we have three options: work backward in the table from (1200, 0) to the point (0, b) on the negative y-axis, use point-slope, or substitute a point and the slope into y = mx + b.

Option 1: Work backward in the table from (1200, 0) to the point (0, b) on the negative y-axis.

x	*y*
0	–3960
100	–3630
200	–3300
300	–2970
400	–2640
500	–2310
600	–1980
700	–1650
800	–1320
900	–990
1000	–660
1100	–330
1200	0

The y-intercept is –3960. Therefore, the second equation is y = 3.3x – 3960.

Option 2: Using point-slope to find the y-intercept. We can choose the point (900, –990).

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - (-990) &= 3.3(x - 900) \\
y + 990 &= 3.3(x - 900) \\
y - 990 &= 3.3x - 2970 \\
y &= 3.3x - 3960
\end{aligned}
$$

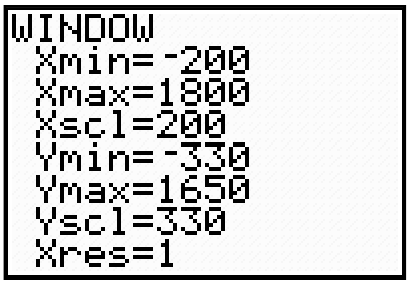
Option 3: Substituting a point, such as (900, –990), and the slope into slope-intercept form and solving for b.

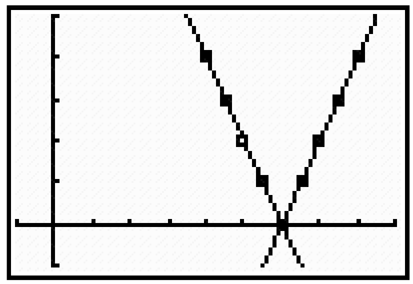
$$
\begin{aligned}
y &= mx + b \\
-990 &= 3.3(900) + b \\
-990 &= 2970 + b \\
-3960 &= b \\
y &= 3.3x - 3960
\end{aligned}
$$

10. Graph the equations on your calculator. How are the equations similar? How are they different?

The equations are similar because they each have a constant rate of change. The absolute value of the slopes of the equations is 3.3. The absolute value of the y-intercepts of the equations is 3960. The equations are different because the slope of one equation is –3.3 and the slope of the other is 3.3. The y-intercept of one equation is 3960 and the y-intercept of the other is –3960.

11. If necessary adjust the window to clearly see the intersection of the two lines. What does the intersection of these two lines represent? Sketch the graph.





The intersection of the two lines represents the location of the fire station.

12. Where do the equations fit the graph of the data points? Where do the equations not fit the graph of the data points?

The equations fit the data points above the x-axis. The equations do not fit the data points below the y-axis.

13. How well do the linear functions model the data points?

Linear functions model the data points above the point of intersection well. Below the point of intersection, the functions do not fit the data.

14. Write summary statements about the conceptual understanding of the absolute value of a number and linear equations that model a situation using absolute value.

Responses may vary. The conceptual understanding of the absolute value of a number can be modeled through a situation relating address number and distance from a particular location. Linear equations can be used to model an absolute value function, but contain points not in the absolute value function.

<u>Part 2</u>: The Relay Race

Leader Notes:
In this part of the professional development, participants model the concept of the absolute value function by walking toward and away from a motion detector at a constant speed. At the end of this part, participants should be able to model an absolute value function using the CBR and connect the graph of the model to an absolute value function.

Technology Tip:
Participants may not have experience using the CBR. Be prepared to assist participants who have questions regarding the data collection process. You may need to explain to the participants that the Calculator-Based Ranger motion detector (CBR) measures the distance between an object and itself at a specific point in time.

Facilitation Tip:
You will want to have someone read aloud the problem situation at the top of the participant page. You will also want to ask a volunteer to model the relay race before the rest of the group. Do not show them the graph of the function.

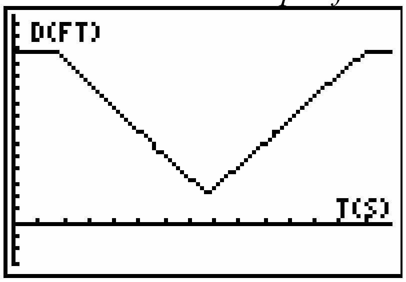
> **Pretend you are participating in an unusual relay race. The object of the race is to walk toward the CBR at a slow steady rate as if to pick up something and then immediately to walk backwards away from the CBR at the same rate without stopping. The person whose rate walking towards the CBR matches the rate walking away from the CBR and who changes direction instantly wins the race!**

Note to Leader: Participants may notice that their rates are not exactly additive inverses of each other. The difference may occur from people's tendency to walk more slowly when they are walking backwards. You may want to suggest that groups collect new data when they actually turn around quickly so that they are walking forward.

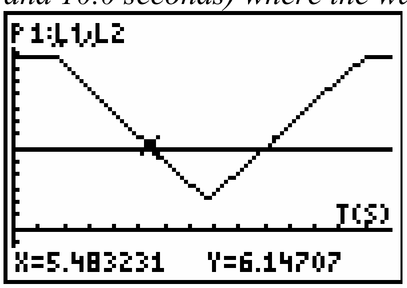
1. **Predict and sketch the distance versus time graph of the volunteer's walk in the space below.**
   Sample sketches may vary. Teachers may have different predictions about the walk.

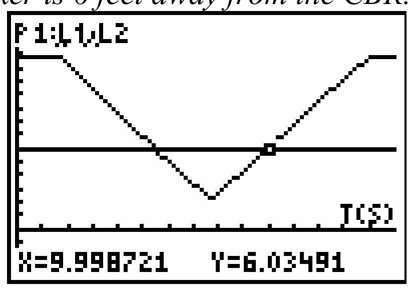
2. **Using the Ranger program on the APPS menu of the calculator, a CBR and a link cable, collect data on the relay race. You may want to move to an area that provides room for you to walk.**

3. **What is the shape of the graph of your walk? How does the graph of the walk compare to your prediction?**
   The walk is in the shape of a "v." A sample walk is shown below. Responses may vary.

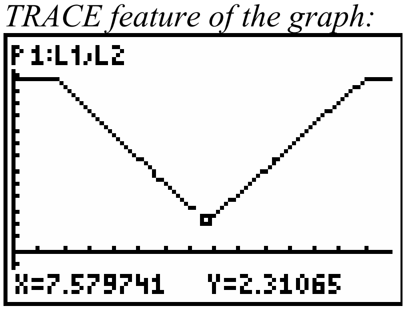


4. **How many times is the walker a given distance from the CBR?**
   At two different times the walker is the same distance from the CBR. The walker is at a given distance from the CBR one time during the walk toward the CBR and a second time on the walk away from the CBR. For example, for the sample data, there are two times (5.5 seconds and 10.0 seconds) where the walker is 6 feet away from the CBR.

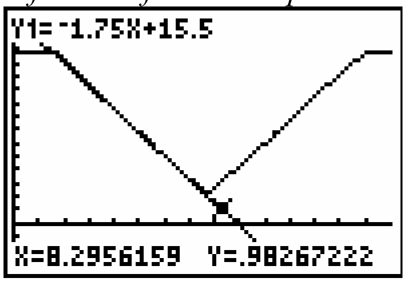




5. **At what point on the graph does the direction of the walk change? How can you interpret this point in terms of the time and distance?**
   Responses may vary. In the sample graph, the point is (7.57, 2.31). At about 7.57 seconds, the walker was about 2.31 feet from the CBR when he/she changed direction. Using the TRACE feature of the graph:

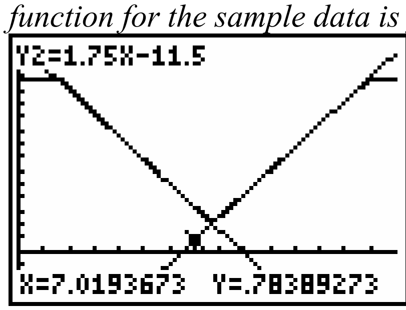


6. What part of the graph represents your motion toward the CBR? What function rule best describes the walk toward the CBR?
Responses may vary. The first part of the graph (from when the walker began the walk to the point that he/she reversed direction) represents the motion toward the CBR.
A function for the sample data is $y = -1.75x + 15.5$.



7. What is the domain for this part of the walk? How does this domain compare to the domain of the function?
Responses may vary. The domain for the sample data is $1.77 \le x \le 7.57$. The domain for the function is the set of all real numbers. The domain for the first part of the walk is a subset of the domain of the function.

8. What part of the graph represents motion away from the CBR? What function rule best describes the walk away from the CBR?
Responses may vary. The second part of the graph (from when the walker reversed direction to the point that the CBR stopped collecting data) represents motion away from the CBR. A function for the sample data is $y = 1.75x - 11.5$.



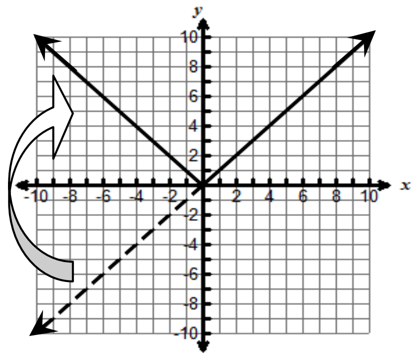
9. What is the domain for this part of the walk? How does this domain compare to the domain of the function?
Responses may vary. The domain for the sample data is $7.57 \le x \le 12.92$. The domain for the function is the set of all real numbers. The domain for the second part of the walk is a subset of the domain of the function.

10. How do the functions compare? Does this match your expectation? If there are differences, what might explain them?
Responses may vary. The equations in the sample data are not "opposites" of each other. The rate walking toward the CBR, –1.75 feet per second, is the additive inverse, or "opposite" of the rate walking away from the CBR, 1.75 feet per second.

11. How can we write one function rule that describes the entire walk?
We can write an absolute value function to describe the entire walk.

12. How does this function remedy domain restrictions we encountered by using two linear functions?

The absolute value function does not require domain restrictions, because the definition of absolute value produces a vertical reflection of the graph for x-values less than 7.57.



13. What is the parent function for absolute value?

The parent function is $f(x) = |x|$.

14. What are the characteristics of absolute value functions?

Absolute value functions graph in the shape of a "v." The slope of the left side of the graph is the opposite of the slope of the right side of the graph. The graph has line symmetry through a point (the vertex).The absolute value parent function is produced by reflecting across the x-axis the part of the graph of $y = x$ to the left of $x = 0$.

15. How can we use what we know about the linear functions we wrote to write one absolute value function that fits the graph of the walk? Explain your answer.

Responses may vary. We can rewrite the function $y = -1.75x + 15.5$ as $y = 1.75(-x) + 15.5$. By combining that function with the function $y = 1.75x - 11.5$, we can write the absolute value function $y = 1.75|x|$. The point on the graph,(7.57, 2.31),where the direction of the walk changes represents the vertex of the absolute value function. The function for the sample is $y = 1.75|x - 7.57| + 2.31$.

16. Write a summary statement about how modeling an absolute value function through an activity such as The Relay Race connects real-life situations to Algebra II concepts.

Responses may vary. Modeling an absolute value function allows students to connect motion and time versus distance graphs to Algebra II functions. Through modeling, students make connections among physical, graphical, and symbolic representations.

<u>Part 3</u>: Transformations of the Parent Function

Leader notes:
In this part of the professional development, participants investigate transformations of the absolute value parent function through parameter changes of *a, h,* and *k*. Ask teachers to complete the tables in this section. One of the multiple representations of an absolute value function is given in each row of the table. You may need to assist participants who have questions. At the end of this part, participants should be able to connect the symbolic, graphical, tabular, and verbal representations of absolute value functions and relate the given function to the parent function.

Use the following facilitation questions as needed to assist teachers in completing the multiple representations of **Transformations on the Parent Function**.

Facilitation Questions:

- If you are given an equation, how can you use the graphing calculator to assist you in sketching the graph or completing the table?
  You can write the equation in y= and use the graph and table features of the calculator.

- If you are given a graph, how can you use the graphing calculator to assist you in completing the table or finding the equation?
  You can use the STAT feature to plot the points and then find the equation through trial and error. You can also compare the coordinates of the data points to the coordinates of the parent function to predict the changes to the parent function and the resulting change to the parent function.

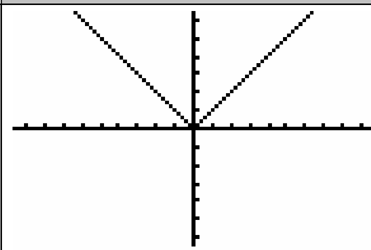
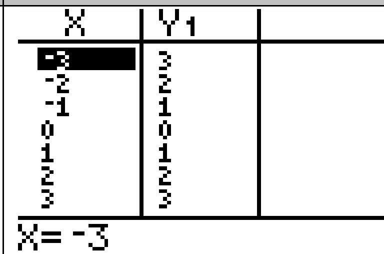
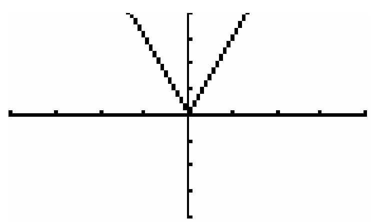
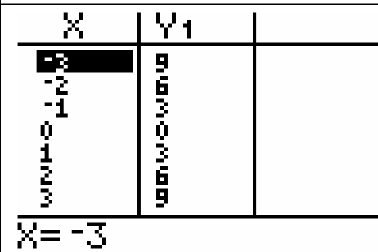
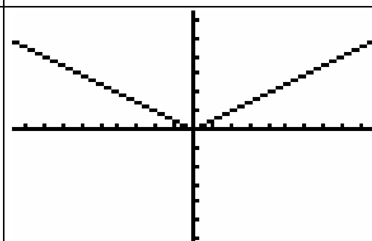
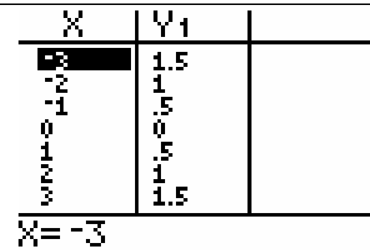
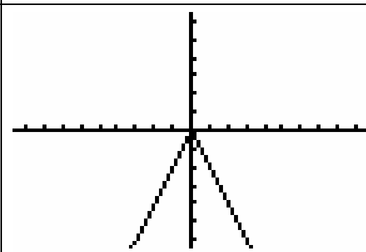
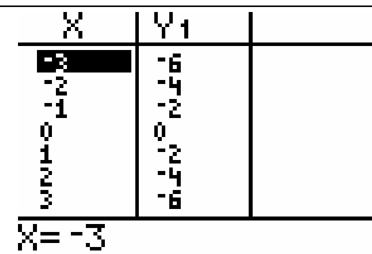
- If you are given a table, how can you use the graphing calculator to assist you in sketching the graph or finding the equation?
  You can use the STAT feature to plot the points and then find the equation through trial and error. You can also compare the coordinates of the data points to the coordinates of the parent function to predict the changes to the parent function and the resulting change to the parent function.

- How can you write a verbal description of the effect on the parent function?
  You can relate the resulting graph to the graph of the parent function in terms of vertical stretches and compressions, reflections, and horizontal and vertical shifts.

- If you are given the verbal description of the effect on the parent function, how can you use what you know about transformations to assist you in writing the equation, sketching the graph, or making a table?
  You can use what you know about transformations to write an equation and compare its graph to the graph of the parent function.

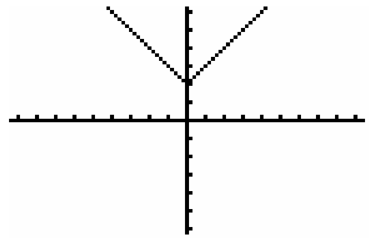
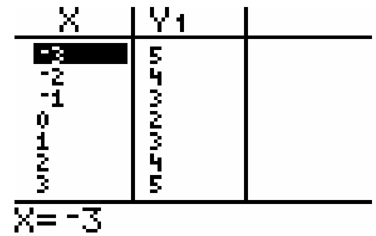
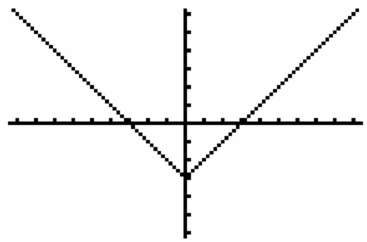
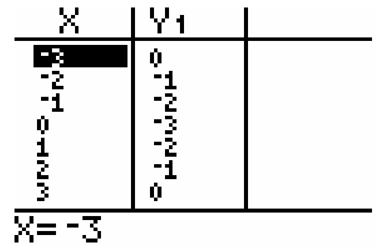
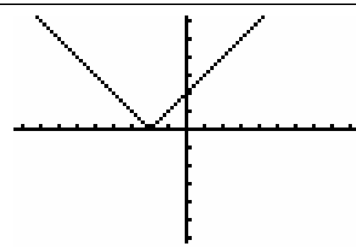
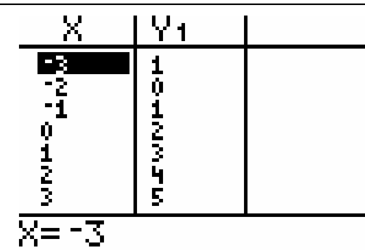
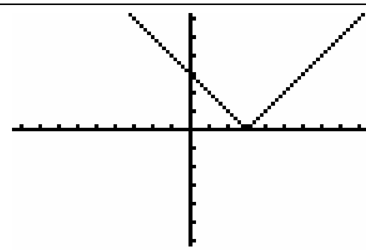
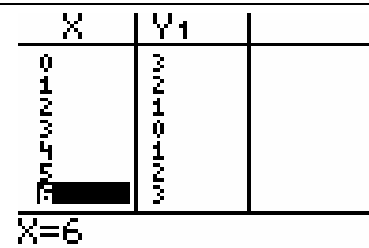
- What window can you use to get a graph that is similar to the one in the table?
  (–9.4, 9.4, 1, –6.2, 6.2, 1,1)

Transformations of the Parent Function: Changes to *a*

Function	Graph	Table	Effect on Parent Function
$y = \lvert x \rvert$			*There is no effect on the parent function.*
$y = 3\lvert x \rvert$			*The y-values of the parent function are multiplied by a factor of 3. (There is a vertical stretch in the parent function by a factor of 3.)*
$y = \frac{1}{2}\lvert x \rvert$			*The y-values of the parent function are multiplied (vertically compressed) by a factor of* $\frac{1}{2}$.
$y = -2\lvert x \rvert$			The y-values of the parent function have been multiplied by a factor of –2, which vertically stretches the graph of the parent function and reflects it across the *x*-axis.

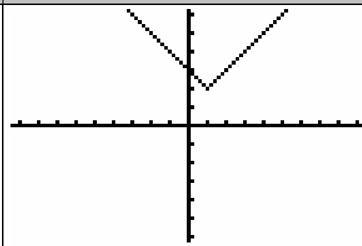
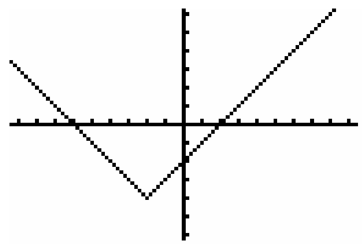
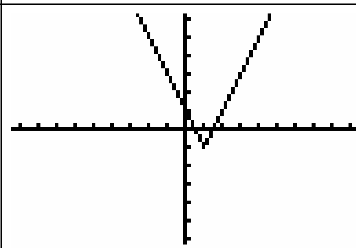
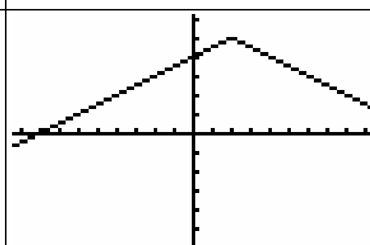
Write a generalization about how the changes in the parameter *a* affect the graph of the parent function.
The parameter a vertically stretches or compresses the graph of the parent function. If a is positive, the graph has the same orientation as the parent function. If a is negative, the graph is reflected across the x-axis.

Transformations of the Parent Function: Changes to *h* and *k*

Function	Graph	Table	Effect on Parent Function
$y = \|x\| + 2$			*The y-coordinate of the vertex is increased by 2 units.* *The graph of the parent function is shifted vertically by 2 units.*
$y = \|x\| - 3$			*The y-coordinate of the vertex is decreased by 3 units.* *The graph of the parent function is shifted vertically by –3 units.*
$y = \|x + 2\|$			The *x*-coordinate of the vertex is moved to the solution for $x + 2 = 0$. The graph of the parent function is shifted to the left by 2 units.
$y = \|x - 3\|$			*The x-coordinate of the vertex is moved to the solution for* $x - 3 = 0$. *The graph of the parent function is shifted to the right by 3 units.*

Write a generalization about how changes in the parameters *h* and *k* affect the graph of the parent function.
The parameter k shifts the parent function vertically such that k is the y-coordinate of the vertex. The parameter h shifts the parent function horizontally so that h is the x-coordinate of the vertex.

Transformations of the Parent Function: Changes to a, *h,* and *k*

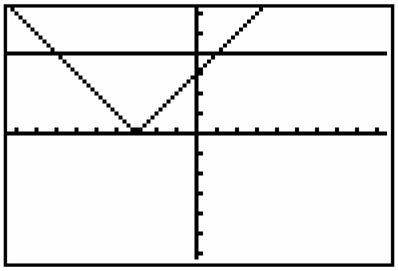
Function	Graph	Table	Effect on Parent Function
$y = \lvert x-1 \rvert + 2$		X: -3, -2, -1, 0, 1, 2, 3 Y1: 6, 5, 4, 3, 2, 3, 4 X= -3	*The vertex of the parent function has been translated to (1, 2).*
$y = \lvert x+2 \rvert - 4$		X: -3, -2, -1, 0, 1, 2, 3 Y1: -3, -4, -3, -2, -1, 0, 1 X= -3	The vertex of the parent function has been translated to (–2, –4).
$y = 2\lvert x-1 \rvert - 1$		X: -3, -2, -1, 0, 1, 2, 3 Y1: 7, 5, 3, 1, -1, 1, 3 X= -3	*The parent function has been stretched vertically by a factor of 2 and the vertex has been translated to (1,–1).*
$y = -\frac{1}{2}\lvert x-2 \rvert + 5$		X: -3, -2, -1, 0, 1, 2, 3 Y1: 2.5, 3, 3.5, 4, 4.5, 5, 4.5 X= -3	*The parent function has been compressed vertically by a factor of* $-\frac{1}{2}$ *and the vertex has been translated to (2, 5).*

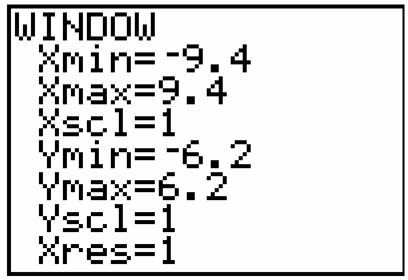
Write a generalization about how changes in the parameters *a, h, and k* affect the graph of the parent function.
The parameter a vertically stretches or compresses the graph of the parent function by scaling the y-values by a factor of a. The vertex of the parent function is shifted to the point (h,k).

<u>Part 4</u>: Solving Absolute Value Equations and Inequalities

Leader Notes:
In this part of the professional development participants investigate connections among the algebraic and graphical solutions of absolute value equations and inequalities.

1. **Consider the system of equations $y = |x+3|$ and $y = 4$. Graph the system and sketch the graph. Describe your viewing window.**



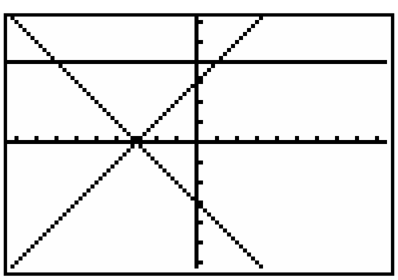


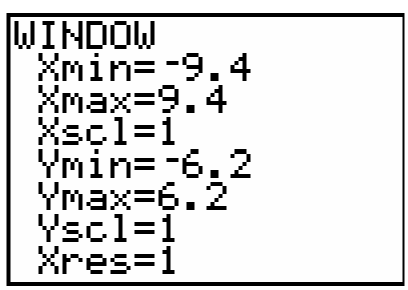
   a) **What are the domain and range for each function in this system?**
   For the function, $y = |x+3|$, the domain is the set of all real numbers and the range is the set of all y-values greater than or equal to 0. For the function, y = 4 ,the domain and is the set of all real numbers, and the range is the set that contains 4.

   b) **What are the coordinates of the points that are solutions for this system? Why are there two solutions?**
   The solutions are (–7, 4) and (1, 4). There are two solutions because the line $y = 4$ intersects $y = |x+3|$ in two points.

   c) **How can you use the concept of substitution to write this system as one equation?**
   $|x+3| = 4$

2. **Graph the functions $y = x+3$, $y = -(x+3)$ and $y = 4$ and sketch the graph. Describe your viewing window.**



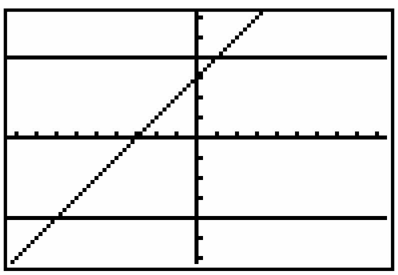


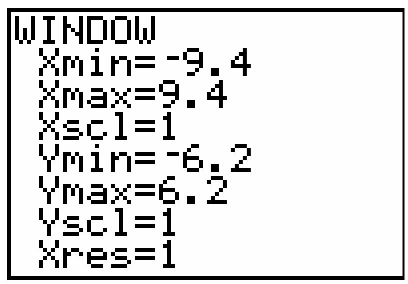
a) What are the domain and range for each function in this system? How does this system of equations compare to the original system?
The domain and range for $y = x + 3$ and $y = -(x + 3)$ is the set of all real numbers. The domain of $y = 4$ is the set of all real numbers and the range is the set that contains 4. The solution for this system is the same as the original system. The absolute value function is represented by parts of the lines $y = x + 3$ and $y = -(x + 3)$.

b) What are the coordinates of the points that are solutions for this system? How do these solutions compare to the solutions of the original system?
The solutions for this system are (–7, 4) and (1, 4). The solution for this system is the same as the original system.

c) How can you use the concept of substitution to write this system of three equations as a system of two equations?
$x + 3 = 4$ *and* $-(x + 3) = 4$

3. Graph the functions $y = x + 3$, $y = 4$, and $y = -4$, and sketch the graph. Describe your viewing window.





a) What are the domain and range for each function in this system?
The domain and range of $y = x + 3$ is the set of all real numbers. The domain of $y = 4$ and $y = -4$ is the set of all real numbers. The range of $y = 4$ is the set that contains 4. The range of $y = -4$ is the set that contains –4.

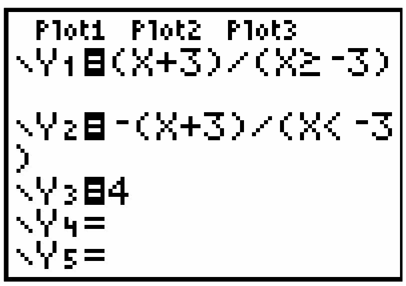
b) What are the coordinates of the points that are solutions for this system? How do the solutions for the graph above compare to the original solutions in question 1?
The solutions for this system are (–7, –4) and (1, 4). Only one solution is the same as the original system, (1, 4).

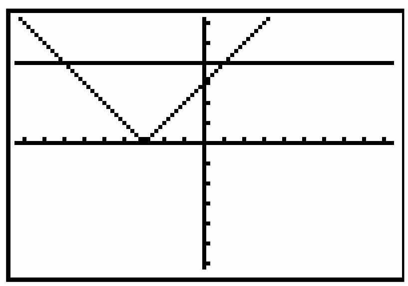
c) Compare the graph above and the graph in question 2 to the graph of the original system. Which graph is conceptually related to the graph of the original system? Which graph is not conceptually related? Why?
The graph in question 2 is conceptually related to the graph of the original system. The graph above is not. The system of equations in question 2 represents the characteristics of the absolute value function and the horizontal line $y = 4$. The system above has two horizontal lines, one of which does not represent the system. Also, the equation $y = x + 3$ only represents half of the absolute value function.

d) What misconceptions might arise by setting up a process to solve $x+3=4$ and $x+3=-4$?
This system is conceptually and graphically different from the original problem and does not represent the intersection of the absolute value function and the line y = 4. Symbolically, the x-coordinates of the solutions are the same, but the y-coordinates are different. So the ordered pairs for this solution do not match the ordered pairs for the original system.

e) What restrictions do we need to place on the domains of the functions $y=x+3$ and $y=-(x+3)$ so that their graphs match the graph of the function, $y=|x+3|$?
The domain of the function $y=x+3$ should be restricted to $x \geq -3$; the domain of the function $y=-(x+3)$ should be restricted to $x < -3$.

f) Graph the system of equations with the restrictions. Sketch your graph.





g) How does the graph of this system of equations and its solutions compare to the graph and solutions to the original system in question 1?
The graphs and solutions are the same.

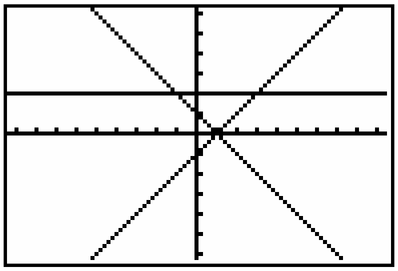
4. Write a system of three equations that conceptually relate to the system of equations $|x-1|=2$.

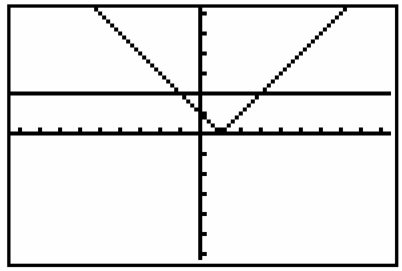
$y = x - 1$

$y = -(x - 1)$

$y = 2$

a) Graph the system you wrote and compare it to the graph of equations $y=|x-1|$ **and** $y=2$ **. How do the graphs compare? How do the solutions compare?**





Parts of the graphs of the linear equations in my system represent the graph of the absolute value function. The solutions are the same.

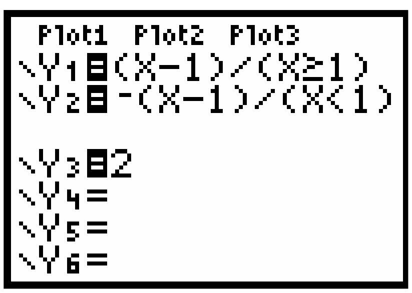
b) How do the equations $x-1=2$ **and** $-(x-1)=2$ **relate to the graphs of the above systems?**

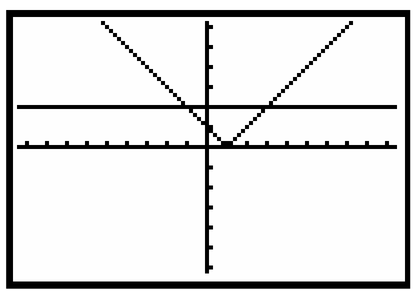
The equations represent the intersection of the absolute value function and the line $y = 2$.

c) What restrictions should we place on the domains of the functions in your system so that the graph of your system matches the graph of $y=|x-1|$ **?**

The domain of the function $y = x-1$ *should be restricted to* $x \geq 1$. *The domain of the function* $y = -(x-1)$ *should be restricted to* $x < 1$.

d) Graph the system of equations with the restrictions. Sketch your graph.





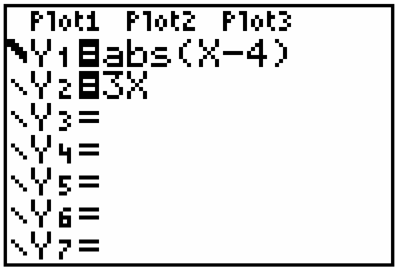
e) How does the graph of this system of equations and its solutions compare to the graph and solutions of $y=|x-1|$ **and** $y=2$ **in question 4a?**

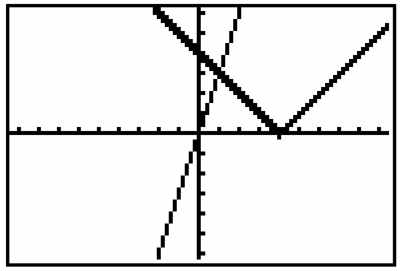
The graphs and solutions are the same.

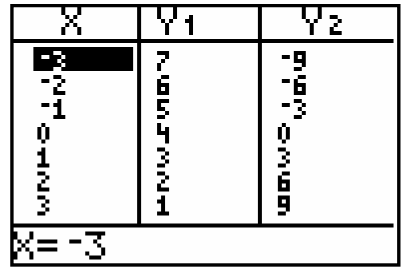
5. Write a statement about the connection between an equation such as $|x+2|=5$ **and the system** $x+2=5$ **and** $-(x+2)=5$ **.**

Responses may vary. The system $x+2=5$ *and* $-(x+2)=5$ *are conceptually the same as* $|x+2|=5$ *and have the same solution set.*

6. Consider the system of equations represented by the equation $|x-4|=3x$. Graph the equations $y=|x-4|$ and $y=3x$, sketch the graph, and complete the table. (Hint: You may want to bold the absolute value equation.)



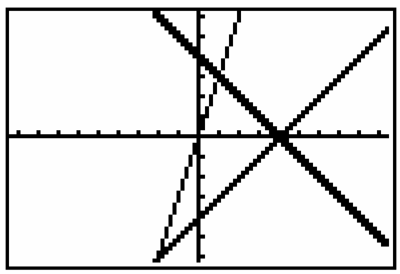
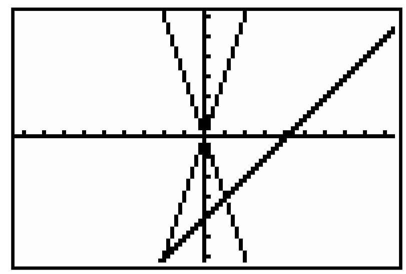




a) How many solutions does this system have? Why?

This system has one solution (1, 3) because the line $y = 3x$ intersects $y=|x-4|$ in only one point.

b) Compare the following systems in the table graphically, tabularly, and algebraically. (Hint: you may want to bold the equation(s) representing the absolute value function.)

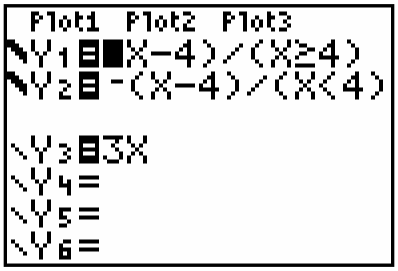
<table>
<tr><th>System A</th><th>System B</th></tr>
<tr><td>$y = x - 4$
$y = -(x-4)$
$y = 3x$</td><td>$y = x - 4$
$y = 3x$
$y = -3x$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>
<table>
<tr><th>x</th><th>y_1</th><th>y_2</th><th>y_3</th></tr>
<tr><td>–3</td><td>–7</td><td>7</td><td>–9</td></tr>
<tr><td>–2</td><td>–6</td><td>6</td><td>–6</td></tr>
<tr><td>–1</td><td>–5</td><td>5</td><td>–3</td></tr>
<tr><td>0</td><td>–4</td><td>4</td><td>0</td></tr>
<tr><td>1</td><td>–3</td><td>3</td><td>3</td></tr>
<tr><td>2</td><td>–2</td><td>2</td><td>6</td></tr>
<tr><td>3</td><td>–1</td><td>1</td><td>9</td></tr>
</table>
</td><td>
<table>
<tr><th>x</th><th>y_1</th><th>y_2</th><th>y_3</th></tr>
<tr><td>–3</td><td>–7</td><td>–9</td><td>9</td></tr>
<tr><td>–2</td><td>–6</td><td>–6</td><td>6</td></tr>
<tr><td>–1</td><td>–5</td><td>–3</td><td>3</td></tr>
<tr><td>0</td><td>–4</td><td>0</td><td>0</td></tr>
<tr><td>1</td><td>–3</td><td>3</td><td>–3</td></tr>
<tr><td>2</td><td>–2</td><td>6</td><td>–6</td></tr>
<tr><td>3</td><td>–1</td><td>9</td><td>–9</td></tr>
</table>
</td></tr>
<tr><td>
$-(x-4) = 3x$ *or* $x - 4 = 3x$
$-x + 4 = 3x$ $-2x = 4$
$-x = -4$ $x = -2$
$x = 1$ $y = -6$
$y = 3$ $(-2, -6)$
$(1, 3)$
</td><td>
$x - 4 = -3x$ *or* $x - 4 = 3x$
$4x = 4$ $-2x = 4$
$x = 1$ $x = -2$
$x = -3$ $y = -6$
$y = 3$ $(-2, -6)$
$(1, -3)$
</td></tr>
</table>

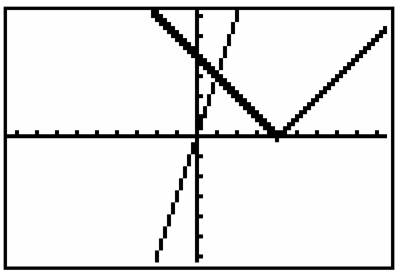
c) What are the solutions for System A? What are the solutions for System B? How do those solutions compare to the original system?

The solutions for System A are (1, 3) and (–2, –6). The solutions for System B are (1, –3) and (–2, –6). Only System A has a solution that is the same solution as the original system.

d) **Which system, A or B, conceptually relates to the original system?**
System A relates conceptually to the original system.

e) **Restrict the domains for the functions in System A, graph the new system, and complete the table. Find the table values for the solutions.**





x	y_1	y_2	y_3
–3	*ER*	*7*	*–9*
–2	*ER*	*6*	*–6*
–1	*ER*	*5*	*–3*
0	*ER*	*4*	*0*
1	*ER*	*3*	*3*
2	*ER*	*2*	*6*
3	*ER*	*1*	*9*

f) How are the solutions for System A without the restricted domain and System A with the restricted domain alike and different?
Both systems have the solution (1, 3). System A with an unrestricted domain has as extra solution (–2, –6).

g) Why does System A without the restricted domain produce two solutions? Which solution of A is not a solution of the original system? What do we call that solution?
System A produces two solutions through the algebraic process. The solution (–2, –6) is not a solution of the original system. We call this solution an extraneous root.

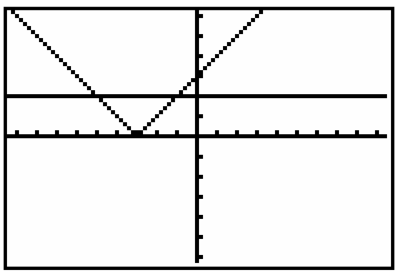
7. What understanding does graphing a system involving absolute value equations provide with regard to the actual number of solutions to the system and the corresponding equations that intersect? How does the graphical solution connect to the algebraic process?
By graphing the system, we can see how many roots the system has. We can also see where the solution is located and the corresponding equations that intersect. By graphing the system first, we can see which equations intersect to form the solutions. We can also avoid extraneous roots.

8. Write a statement comparing the common algebraic process for solving absolute value equations to the conceptual understanding of the solutions to a system of absolute value equations.

The common algebraic process of setting the principal part of the absolute value equal to the positive and negative right-hand side quantities does not align conceptually to the graphical solution of the equation and may give us the same x-values, but not the same y-values. We may also be solving an equation where a root does not exist in the domain (an extraneous root).

9. Consider the system that represents the inequality $|x+3| > 2$ **.**

a) Show the solution graphically, tabularly, and symbolically.



X	Y1	Y2
-6	3	2
-5	2	2
-4	1	2
-3	0	2
-2	1	2
-1	2	2
0	3	2

X= -6

$x \geq -1$ *or* $x \leq -5$

b) When we ask students to show us where $|x+3| > 2$ **, what are we asking?**

We are asking students to find the x-values where the corresponding y-values for the function $y = |x+3|$ *are greater than the y-values of* $y = 2$.

Explain

Leaders' Note: The Maximizing Algebra II Performance (MAP) professional development is intended to be an extension of the ideas introduced in Mathematics TEKS Connections (MTC). Throughout the professional development experience, we allude to components of MTC such as the Processing Framework Model, the emphasis of making connections among representations, and the links between conceptual understanding and procedural fluency.

Debriefing the Experience:

1. **What concepts did we explore in the previous set of activities? How were they connected?**
   Responses may vary. Participants should observe that investigating the physical and graphical representations of absolute value functions enhances understanding of the concept of absolute value. Absolute value equations and inequalities can be solved by graphing the equations and inequalities as systems.

2. **What procedures did we use to describe absolute value functions? How are they related?**
   Tabular, graphical, and symbolic procedures were all used throughout the Explore phase. Ultimately, they are all connected through the numerical relationships used to generate them.

3. **What knowledge from Algebra I do students bring about absolute value functions?**
   Absolute value is not a student expectation until Algebra II, so students may not have had prior instruction on the absolute value of a number or absolute value functions.

4. **After working with absolute value functions in Algebra II, what are students' next steps in Precalculus or other higher mathematics courses?**
   According to the Precalculus TEKS, students will expand their understanding of absolute value functions. They will be expected to apply basic transformations and compositions with absolute value functions, including $\left|f\left(x\right)\right|$, *and* $f\left(\left|x\right|\right)$, *to the parent functions.*

Anchoring the Experience:

5. **Distribute to each table group a poster-size copy of the Processing Framework Model.**

6. **Ask each group to respond to the question:**

   Where in the processing framework would you locate the different activities from the Explore phase?

7. **Participants can use one color of sticky notes to record their responses. In future Explain phases, participants will use other colors of sticky notes to record their responses.**

Horizontal Connections within the TEKS

8. **Direct the participants' attention to the second layer in the Processing Framework Model: Horizontal Connections among Strands.**

9. **Prompt the participants to study the Algebra II TEKS and record those TEKS that connect to this Explore/Explain cycle. Prompt participants to attend to both the knowledge statements as well as the student expectations.**

10. **Invite each table group to share 2 connections that they found and record them so that they are visible to the entire group.**

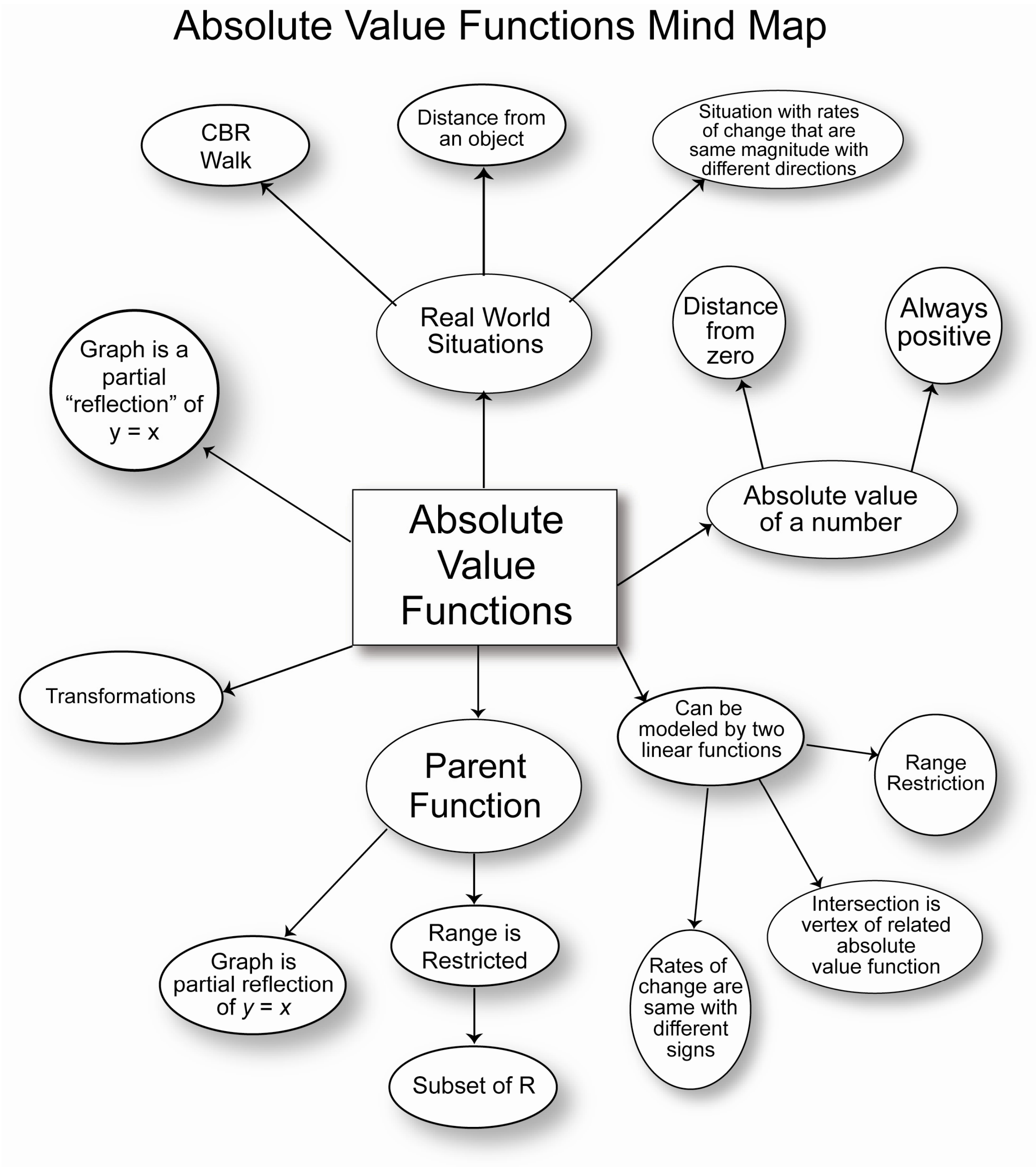
Vertical Connections within the TEKS

11. **Direct the participants' attention to the third layer in the Processing Framework Model: Vertical Connections across Grade Levels.**

12. **Prompt the participants to study the Algebra I, Geometry, Math Models, and Precalculus TEKS and record those TEKS that connect to this Explore/Explain cycle. Prompt participants to attend to both the knowledge statements as well as the student expectations.**

13. **Invite each table group to share 2 connections that they found, recording so that the entire large group may see.**

14. **Provide each group of participants with a clean sheet of chart paper. Ask them to create a "mind map" for the mathematical term of "absolute value."**



15. **Provide an opportunity for each group to share the mind maps with the larger group. Discuss similarities, differences, and key points brought forth by participants.**

16. **Distribute the vocabulary organizer template to each participant. Ask participants to construct a vocabulary model for the term "absolute value functions."**

17. **When participants have completed their vocabulary models, ask participants to identify strategies from their experiences so far in the professional development that could be used to support students who typically struggle with Algebra II topics.**

    Note to Leader: *You may wish to have each small group brainstorm a few ideas first, then share their ideas with the large group while you record their responses on a transparency or chart paper.*

18. **How would this lesson maximize student performance in Algebra II for teaching and learning the mathematical concepts and procedures associated with absolute value functions?**
    Responses may vary. Anchoring procedures within a conceptual framework helps students understand what they are doing so that they become more fluent with the procedures required to accomplish their tasks. Problems present themselves in a variety of representations; providing students with multiple procedures to solve a given problem empowers students to solve the problem more easily.

Elaborate

Leaders' Note: *In this phase, participants will extend their learning experiences to their classroom.*

1. **Distribute the 5E Student Lesson planning template. Ask participants to think back to their experiences in the Explore phase. Pose the following task:**

   What might a student-ready 5E lesson on absolute value functions look like?
   - ❑ **What would the Engage look like?**
   - ❑ **Which experiences/activities would students explore firsthand?**
   - ❑ **How would students formalize and generalize their learning?**
   - ❑ **What would the Elaborate look like?**
   - ❑ **How would we evaluate student understanding of inverses of relations/functions?**

2. **After participants have recorded their thoughts, direct them to the student lesson for absolute value functions. Allow time for participants to review lessons.**

3. **How does this 5E lesson compare to your vision of a student-centered 5E lesson?**
   Responses may vary.

4. **How does this lesson help remove obstacles that typically keep students from being successful in Algebra II?**
   By connecting the concept of absolute value to a physical model, students gain a better understanding of the absolute value function and the limitations of using linear function to describe the model. By solving absolute value equations as systems and through graphing, solutions can be verified and connected to the algebraic processes commonly used to find those solutions. Students can use alternate methods with which to solve meaningful problems.

5. **How does this lesson maximize your instructional time and effort in teaching Algebra II?**
   Taking time to create a solid conceptual foundation reduces the need for re-teaching time and effort and increases student participation in the learning process. Conceptual connections to algebraic process strengthen the understanding of absolute value functions and mirror the links among multiple representations.

6. **How does this lesson maximize student learning in Algebra II?**
   Using multiple representations and foundations for functions concepts allows students to make connections among different ideas. These connections allow students to apply their learning to new situations more quickly and readily.

7. **How does this lesson accelerate student learning and increase the efficiency of learning?**
   Foundations for functions concepts such as function transformations transcend all kinds of functions. A basic toolkit for students to use when working with functions allows students to rethink what they know about linear and quadratic functions while they are learning concepts and procedures associated with other function families.

8. Read through the suggested strategies on Strategies that Support English Language Learners. Consider the possible strategies designed to increase the achievement of English language learners.

As participants read through the strategies that support English language learners and strategies that support students with special needs, they may notice that eight of the ten strategies are the same. The intention is to underscore effective teaching practices for all students. However, English language learners have needs specific to language that students with special needs may or may not have. The two strategies that are unique to the English language learners reflect an emphasis on language. Students with special needs may have prescribed modifications and accommodations that address materials and feedback. Students with special needs often benefit from progress monitoring with direct feedback and adaptation of materials for structure and/or pacing. A system of quick response is an intentional plan to gather data about a student's progress to determine whether or not the modification and (or) accommodation are (is) having the desired effect. The intention of the strategies is to provide access to rigorous mathematics and support students as they learn rigorous mathematics.

9. What evidence of these strategies do you find in this portion of the professional development?
Responses may vary. ***Note:*** *Some strategies reflect teacher behaviors. The presenter may need to prompt participants to consider how the professional development materials support the needed teacher behaviors. For example, a student lesson may outline a structured approach for exploration so that the activity is non-threatening. This contributes to the teacher's ability to create an emotionally safe environment for learning. Tools such as the CBR and graphing calculator can be used to communicate about and solve problem situations involving absolute value functions.*

10. Which strategies require adaptation of the materials in this portion of the professional development?
Responses may vary. Most of the strategies are incorporated throughout the materials.

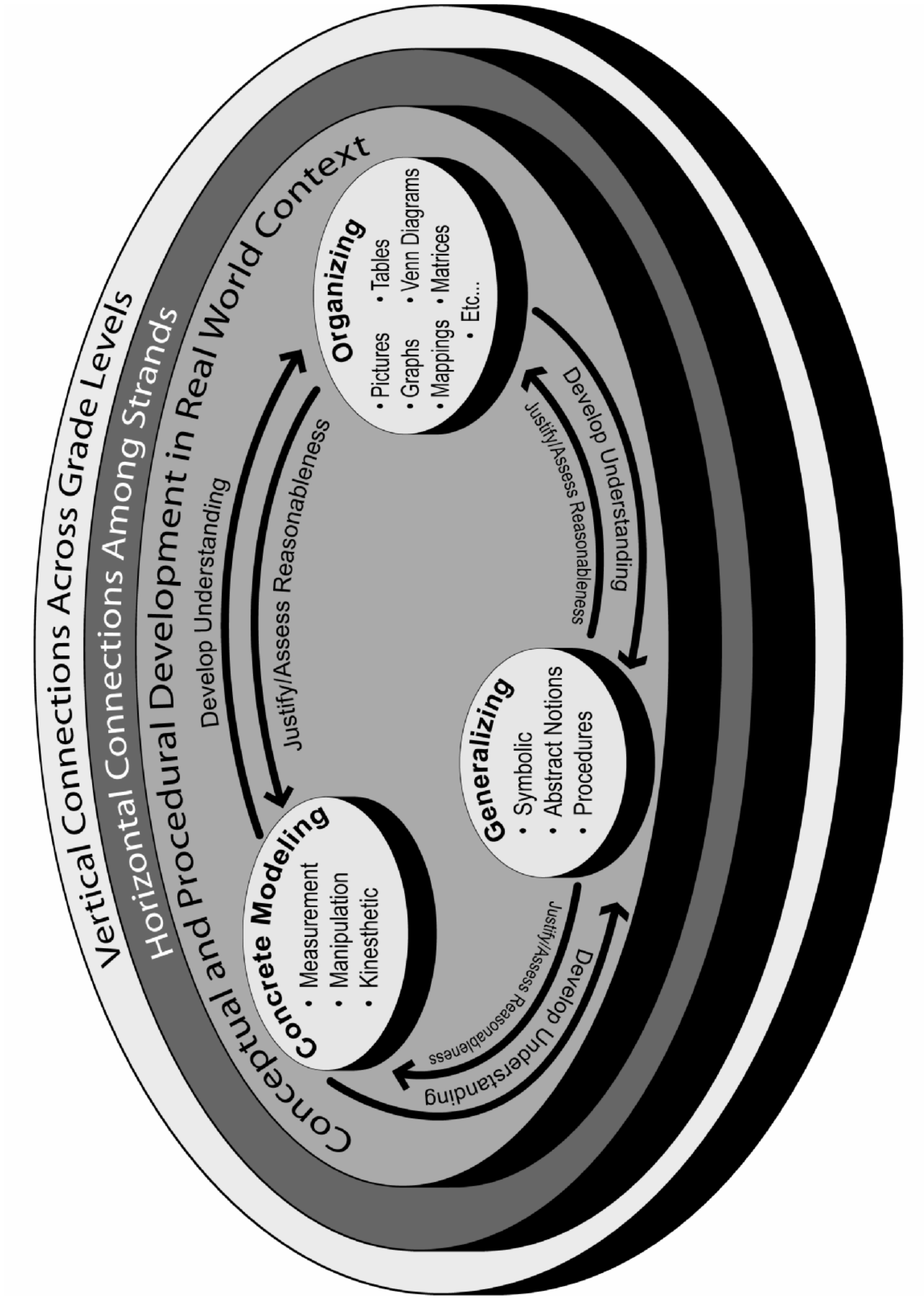
11. Read through the suggested strategies on Strategies that Support Students with Special Needs. Consider the possible strategies designed to increase the achievement of students with special needs.

12. What evidence of these strategies do you find in this portion of the professional development?
Responses may vary. ***Note:*** *Some strategies reflect teacher behaviors. The presenter may need to prompt participants to consider how the professional development materials support the needed teacher behaviors. For example, a student lesson may outline a structured approach for exploration so that the activity is non-threatening. This contributes to the teacher's ability to create an emotionally safe environment for learning. Tools such as the CBR and graphing calculator can be used to communicate about and solve problem situations involving absolute value functions.*

13. Which strategies require adaptation of the materials in this portion of the professional development?

Responses may vary. Most of the strategies are incorporated throughout the materials. Some materials may need to be modified for format.

Processing Framework Model



Vocabulary Organizer

My definition	**Personal Association**
Example	**Non-Example**

5E Lesson Planning Template

Description	Activity
Engage The activity should be designed to generate student interest in a problem situation and to make connections to prior knowledge. The instructor initiates this stage by asking meaningful questions, posing a problem to be solved, or by showing something intriguing.	
Explore The activity should provide students with an opportunity to become actively involved with the key concepts of the lesson through a guided exploration requiring them to probe, inquire, and question. The instructor actively monitors students as they interact with each other and the activity.	
Explain Students collaboratively begin to sequence events/facts from the investigation and communicate these findings to each other and the instructor. The instructor, acting in a facilitation role, formalizes student findings by providing further explanations and additional meaning or information, such as correct terminology.	
Elaborate Students extend, expand, or apply what they have learned in the first three stages and connect this knowledge with prior learning to deepen understanding. Instructors can use the Elaborate stage to verify students' understandings.	
Evaluate Evaluation occurs throughout students' learning experiences. More formal evaluation can be conducted at this stage. Instructors can determine whether the learner has reached the desired level of understanding the key ideas and concepts.	

Strategies that Support English Language Learners (ELL)

Strategy	Explore, Explain, Elaborate 3
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Be conscious of tone and diction. Speak slowly and distinctly.	
Incorporate language skills (reading, writing, speaking, and listening) into instruction.	

Strategies that Support Students with Special Needs

Strategy	Explore, Explain, Elaborate 3
Create an emotionally safe environment for learning.	
Provide ample time for students to process questions before expecting them to respond	
Encourage students to work together in structured activity.	
Link learning to prior knowledge.	
Teach conceptual vocabulary with organizers, concept mappings, and word walls.	
Use a variety of instructional techniques including manipulatives and multiple representations.	
Use scaffolding techniques to build from simple understandings to complex understandings, making conceptual and procedural connections explicit.	
Prompt student to compare and contrast concepts, procedures, and generalizations.	
Use a system of quick response to needs and accommodations including progress monitoring to inform instruction.	
Accommodate materials for format, structure, sequence, etc. as needed.	

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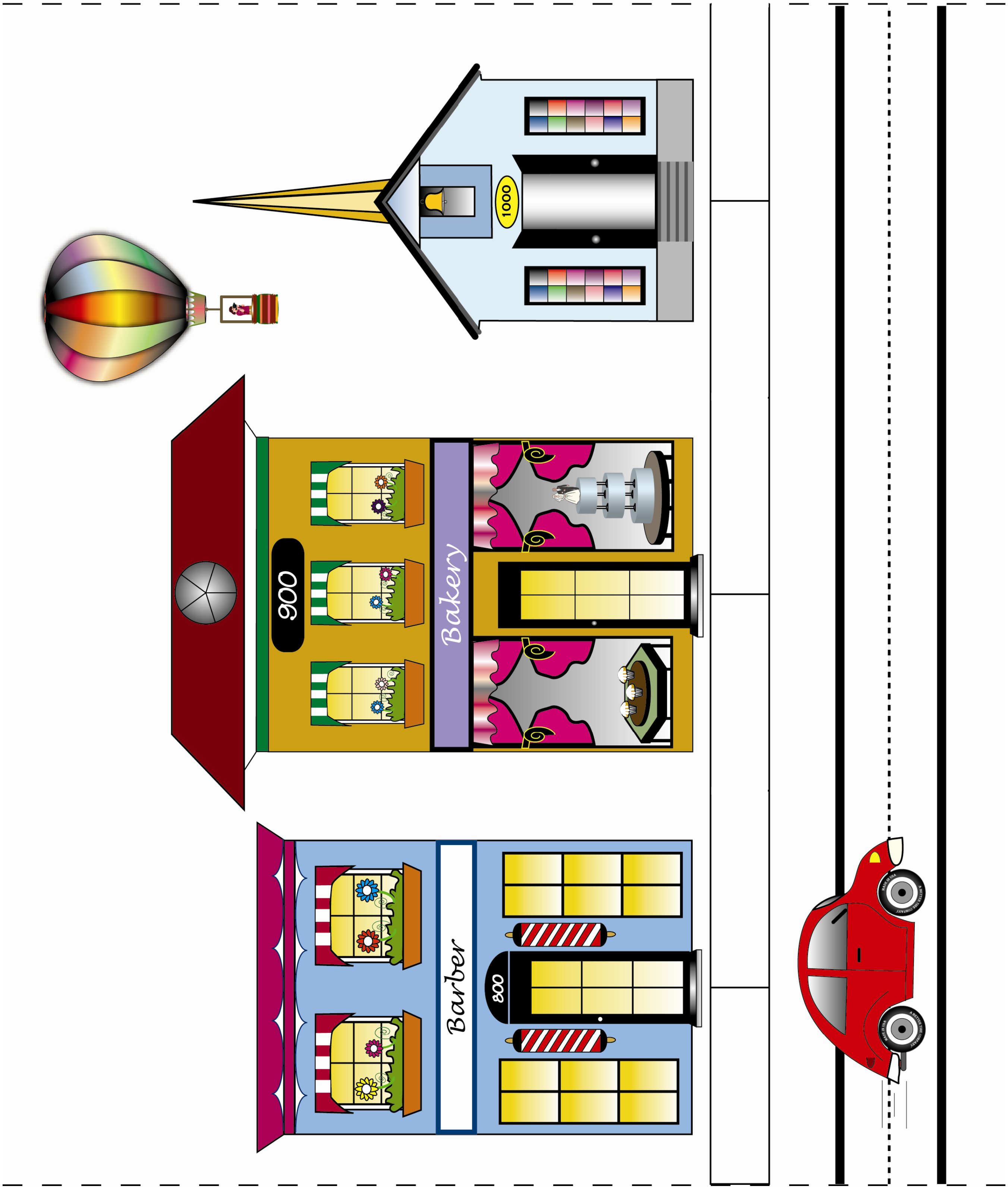

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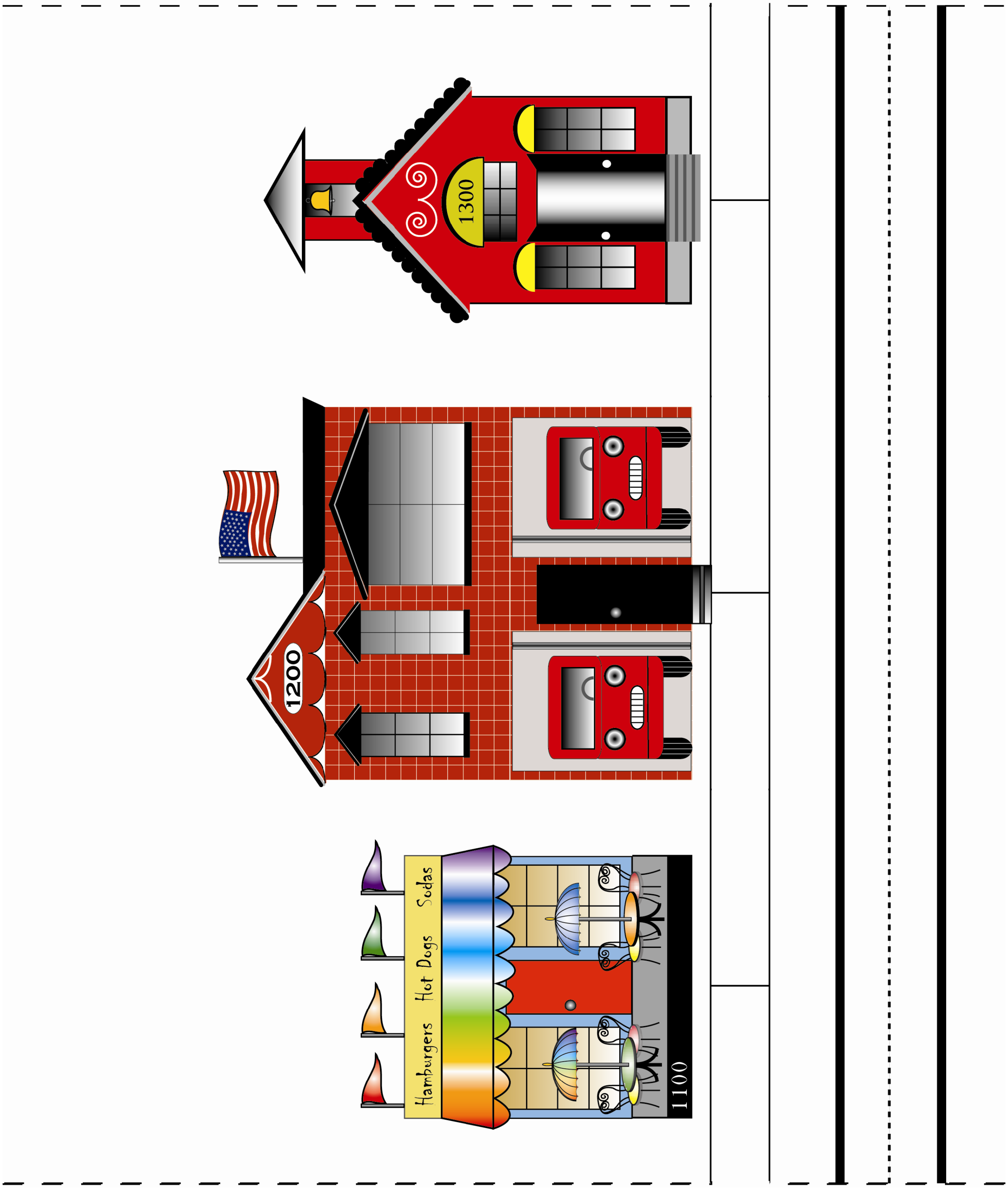
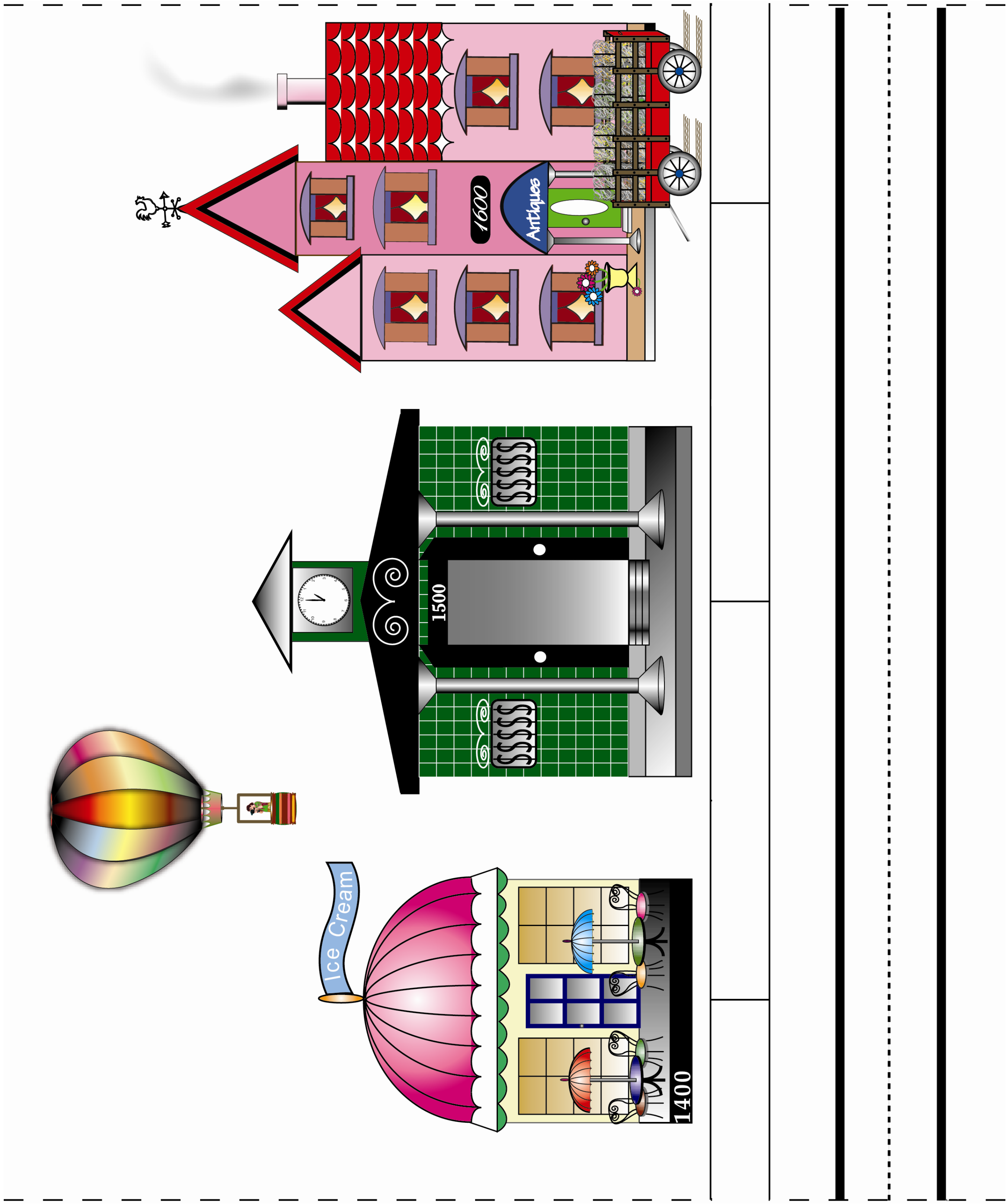



1600
Antiques
1500
Ice Cream
1400

Participant Pages: The Fire Station Problem

A fire station is located on Main Street and has buildings at every block to the right and to the left. You will investigate the relationship between the address number on a building and its distance in feet from the fire station. On average, a mile in the city is composed of 16 city blocks. So each city block is about 330 feet long (5280 feet ÷ 16 = 330 feet). Each building in centered on the block.

1. Complete the table below that relates the address of a building (x) with its distance in feet from the fire station (y).

Address Number (x)	**Distance in Feet from the Fire Station (y)**

2. Which building is 660 feet way from the fire station? Explain your answer.

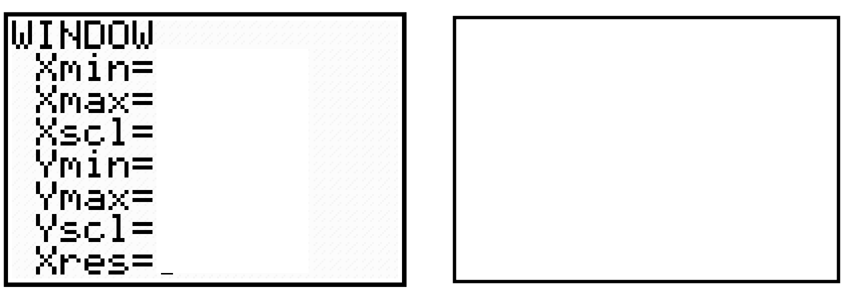
3. If we send someone to the building that is 660 feet away from the fire station, how will she know that she has arrived at the correct place?

4. What words might we use to describe two locations that are the same distance from the fire station?

5. Suppose the buildings on Main Street are renumbered as if they are on a number line so that the location of the fire station represents 0. How do we describe two numbers with the same distance from 0?

6. Draw a scatterplot that represents the data in the table.

7. Make a scatterplot of your data using your graphing calculator. Describe your viewing window.

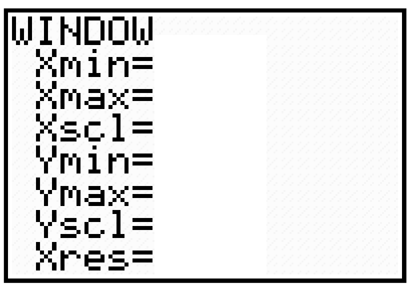


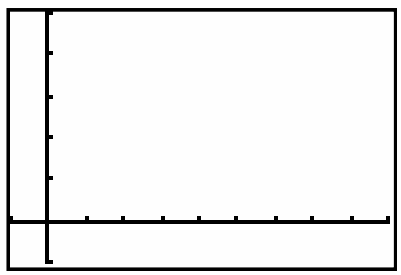
8. What function or functions might students use to describe the scatterplot?

9. Find two linear functions that pass through the data points. What process did you use to find the equations of the lines?

10. Graph the equations on your calculator. How are the equations similar? How are they different?

11. If necessary adjust the window to clearly see the intersection of the two lines. What does the intersection of these two lines represent? Sketch the graph.





12. Where do the equations fit the graph of the data points? Where do the equations not fit the graph of the data points?

13. How well do the linear equations model the data points?

14. Write summary statements about conceptual understanding of the absolute value of a number and linear equations that model a situation using absolute value.

Participant Pages: The Relay Race

Pretend you are in an unusual relay race. The object of the race is to walk toward the CBR at a slow steady rate as if to pick up something and then to walk backwards away from the CBR at the same rate without stopping. The person whose rate walking towards the CBR matches the rate walking away from the CBR and who changes direction instantly wins the race!

1. Predict and sketch the distance versus time graph of the volunteer's walk in the space below.

2. Using the Ranger program on the APPS menu of the calculator, a CBR and a link cable, collect data on the relay race. You may want to move to an area that provides room for you to walk.

3. What is the shape of the graph of your walk? How does the graph of the walk compare to your prediction?

4. How many times is the walker a given distance from the CBR?

5. At what point on the graph does the direction of the walk change? How can you interpret this point in terms of the time and distance?

6. What part of the graph represents your motion toward the CBR? What function rule best describes the walk toward the CBR?

7. What is the domain for this part of the walk? How does this domain compare to the domain of the function?

8. What part of the graph represents motion away from the CBR? What function rule best describes the walk away from the CBR?

9. What is the domain for this part of the walk? How does this domain compare to the domain of the function?

10. How do the functions compare? Does this match your expectation? If there are differences, what might explain them?

11. How can we write one function rule that describes the entire walk?

12. How does this function remedy domain restrictions we encountered by using two linear functions?

13. What is the parent function for absolute value?

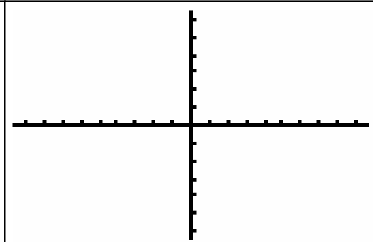
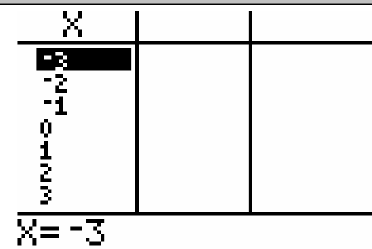
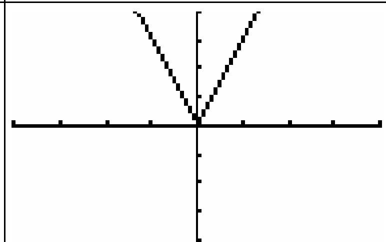
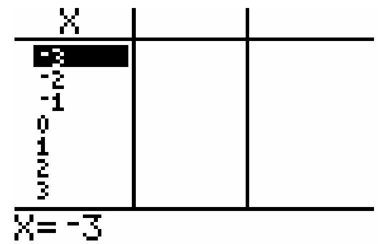
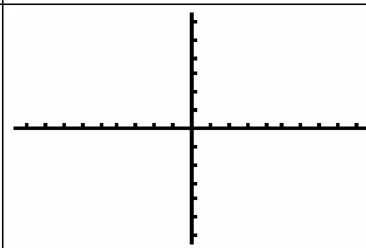
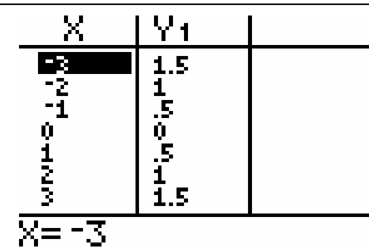
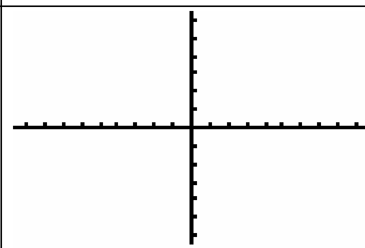
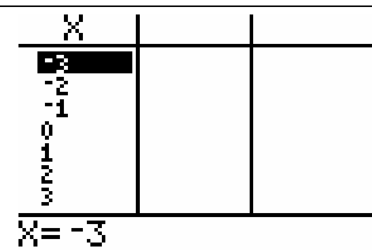
14. What are the characteristics of absolute value functions?

15. How can we use what we know about the linear functions we wrote to write one absolute value function that fits the graph of the walk? Explain your answer.

.

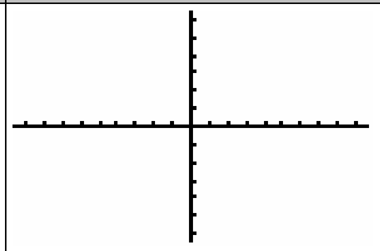
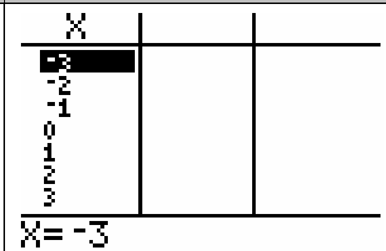
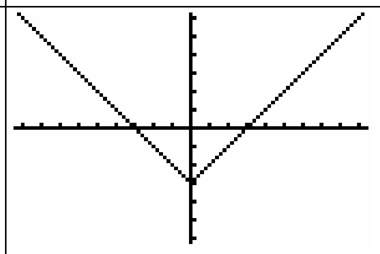
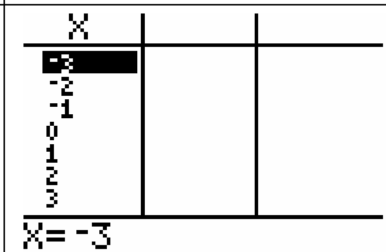
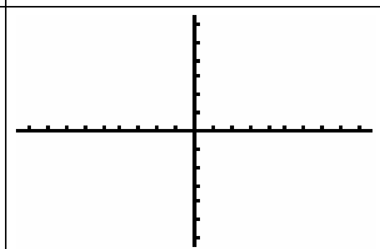
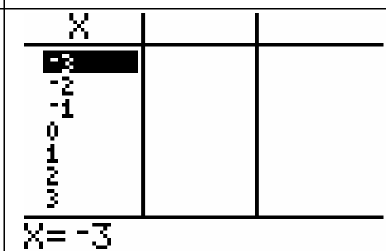
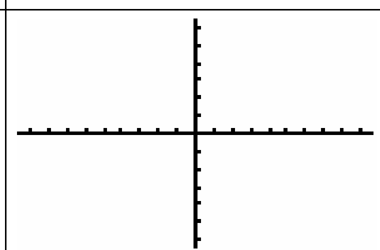
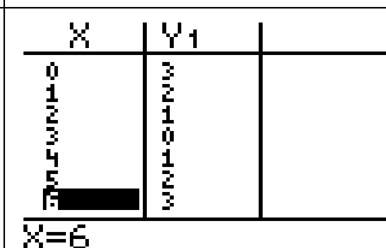
16. Write a summary statement about how modeling an absolute value function through an activity such as The Relay Race connects real-life situations to Algebra II concepts.

Participant Pages: Transformations of the Parent Function: Changes to a

Function	Graph	Table	Effect on Parent Function
$y = \|x\|$			
			
			
			The y-values of the parent function have been multiplied by a factor of –2, which vertically stretches the graph of the parent function and reflects it across the x-axis.

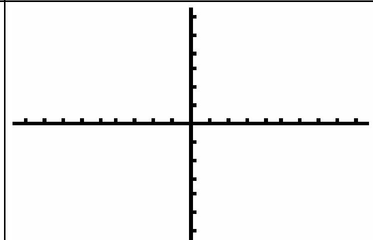
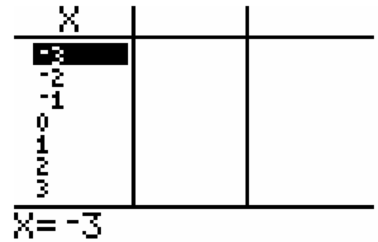
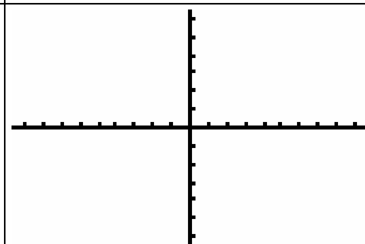
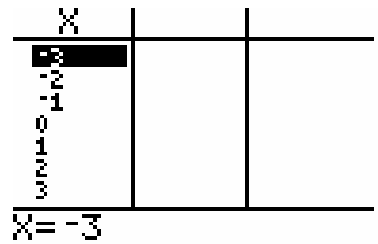
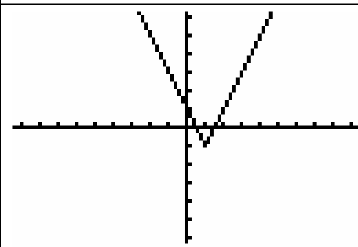
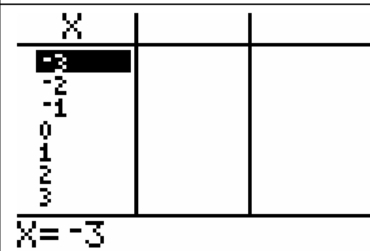
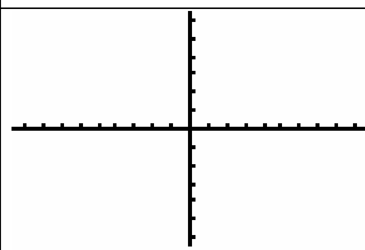
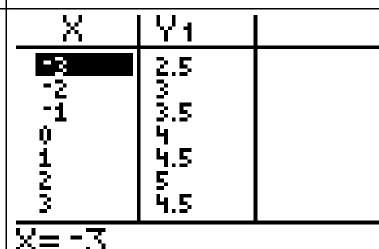
Write a generalization about how the parameter *a* affects the graph of the parent function.

Participant Pages: Transformations on the Parent Function: Changes to *h* and *k*

Function	Graph	Table	Effect on Parent Function
$y=\lvert x\rvert+2$			
			
			The x-coordinate of the vertex is moved to the solution for $x + 2 = 0$. The graph of the parent function is shifted to the left by 2 units.
			

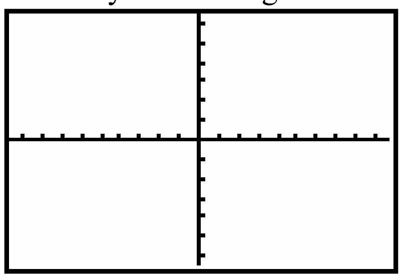
Write a generalization about how changes in the parameters *h* and *k* affect the graph of the parent function.

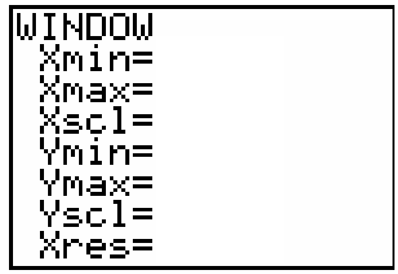
Participant Pages: Transformations to the Parent Functions: Changes to *a*, *h*, and *k*

Equation	Graph	Table	Effect on Parent Function
$y = \lvert x - 1 \rvert + 2$			
			The vertex of the parent function has been translated to (–2, –4).
			
			
Write a generalization about how changes to the parameters *a, h, and k* affect the graph of the parent function.			

Participant Pages: Solving Absolute Value Equations and Inequalities

1. Consider the system of equations $y = |x+3|$ and $y = 4$. Graph the system and sketch the graph. Describe your viewing window.



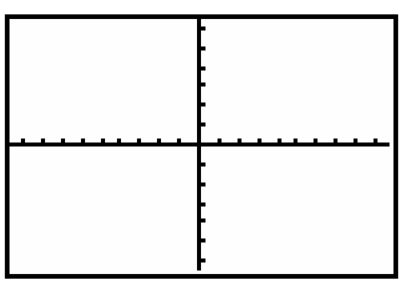


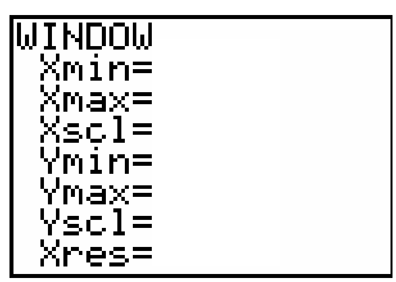
a) What are the domain and range of each function in this system?

b) What are the coordinates of the points that are solutions for this system? Why are there two solutions?

c) How can you use the concept of substitution to write this system as one equation?

2. Graph the functions $y = x+3$, $y = -(x+3)$ and $y = 4$ and sketch the graph. Describe your viewing window.



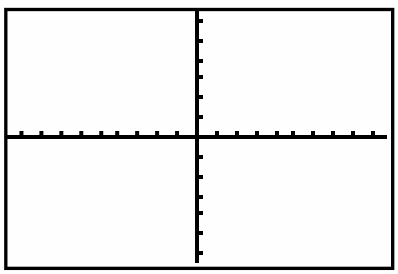


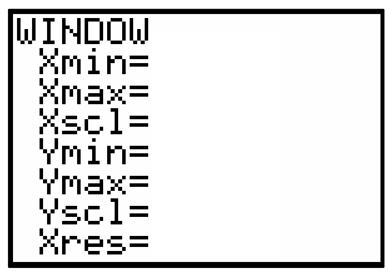
a) What are the domain and range for each function in this system? How does this system of equations compare to the original system?

b) What are the coordinates of the points that are solutions for this system? How do these solutions compare to the solutions of the original system?

c) How can you use the concept of substitution to write this system of three equations as a system of two equations?

3. Graph the functions $y = x + 3$, $y = 4$, and $y = -4$. Graph this system and sketch the graph. Describe your viewing window.





a) What are the domain and range for each function in this system?

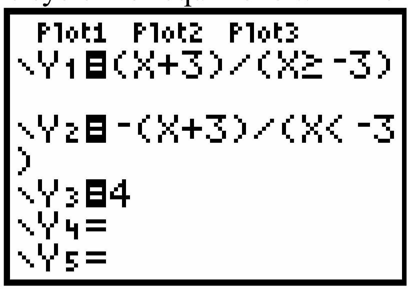
b) What are the coordinates of the points that are solutions for this system? How do the solutions for the graph above compare to the original solutions in question 1?

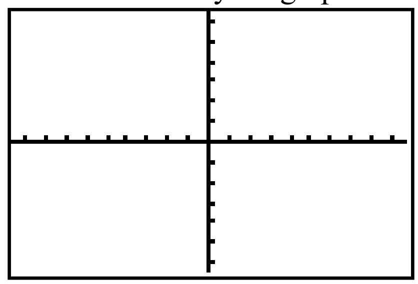
c) Compare the graph above and the graph in question 2 to the graph of the original system. Which graph is conceptually related to the graph of the original system? Which graph is not conceptually related? Why?

d) What misconceptions might arise by setting up a process to solve $x + 3 = 4$ and $x + 3 = -4$?

e) What restrictions do we need to place on the domains of the functions $y = x + 3$ and $y = -(x+3)$ so that their graphs match the graph of the function $y = |x+3|$?

f) Graph the system of equations with the restrictions. Sketch your graph.

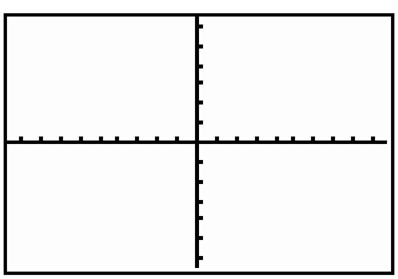


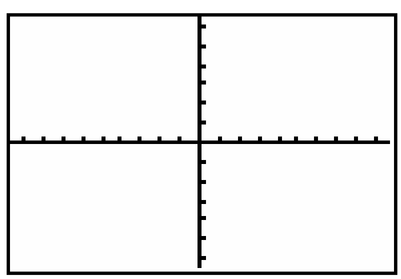


g) How does the graph of this system of equations and its solutions compare to the graph and solutions of $y = |x-1|$ and $y = 2$ in question 4a?

4. Write a system of three equations that conceptually relate to the system of equations $|x-1| = 2$.

a) Graph the system you wrote and compare it to the graph of equations $y = |x-1|$ and $y = 2$. How do the graphs compare? How do the solutions compare?

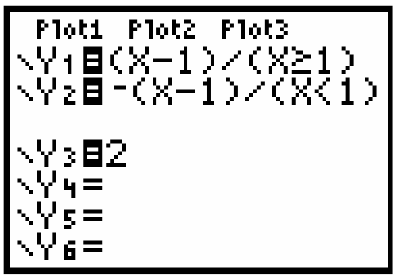


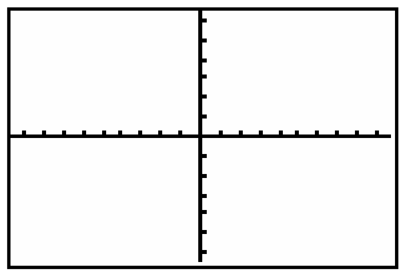


b) How do the equations $x-1=2$ and $-(x-1)=2$ relate to the graphs of the above systems?

c) What restrictions should we place on the domains of the functions in your system so that the graph of your system matches the graph of $y=|x-1|$?

d) Graph the system of equations with the restrictions. Sketch your graph.

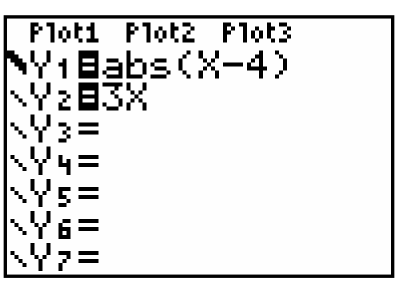


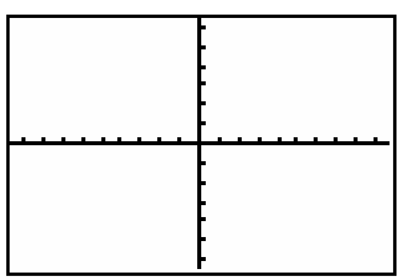


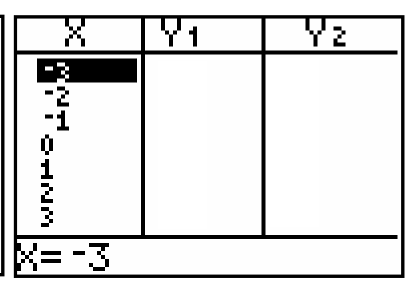
e) How does the graph of this system and its solutions compare to the graph of $y=|x-1|$ and $y=2$ in question 4a?

5. Write a statement about the connection between a system of equations such as $|x+2|=5$ and the system $x+2=5$ and $-(x+2)=5$.

6. Consider the system of equations represented by the equation $|x-4|=3x$. Graph the equations $y=|x-4|$ and $y=3x$, sketch the graph, and complete the table. (Hint: You may want to bold the absolute value equation.)

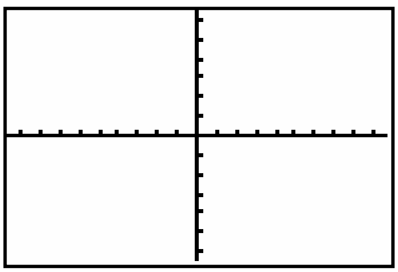
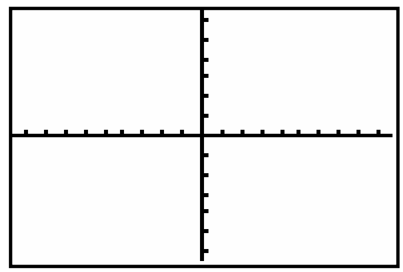






a) How many solutions does this system have? Why?

b) Compare the following systems in the table graphically, tabularly, and algebraically. (Hint: you may want to bold the equation(s) representing the absolute value function.)

<table>
<tr><th>System A</th><th>System B</th></tr>
<tr><td>$y = x - 4$
$y = -(x - 4)$
$y = 3x$</td><td>$y = x - 4$
$y = 3x$
$y = -3x$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>
<table>
<tr><th>x</th><th>y_1</th><th>y_2</th><th>y_3</th></tr>
<tr><td>–3</td><td></td><td></td><td></td></tr>
<tr><td>–2</td><td></td><td></td><td></td></tr>
<tr><td>–1</td><td></td><td></td><td></td></tr>
<tr><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
</table>
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<table>
<tr><th>x</th><th>y_1</th><th>y_2</th><th>y_3</th></tr>
<tr><td>–3</td><td></td><td></td><td></td></tr>
<tr><td>–2</td><td></td><td></td><td></td></tr>
<tr><td>–1</td><td></td><td></td><td></td></tr>
<tr><td>0</td><td></td><td></td><td></td></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
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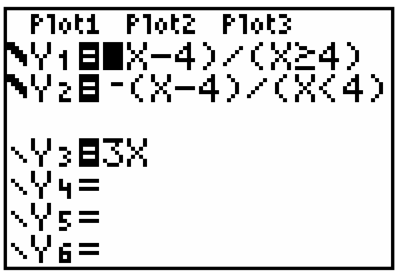
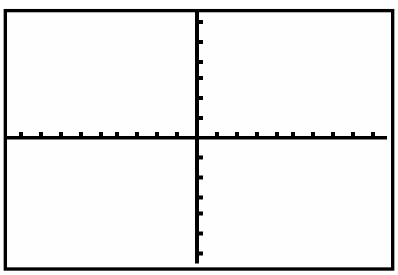
</td></tr>
<tr><td></td><td></td></tr>
</table>

c) What are the solutions for System A? What are the solutions for System B? How do those solutions compare to the original system?

d) Which system, A or B, conceptually relates to the original system?

e) Restrict the domains for the functions in System A, graph the new system and complete the table below. Find the table values for the solutions.





x	y_1	y_2	y_3
–3			
–2			
–1			
0			
1			
2			
3			

f) How are the solutions for System A without the restricted domain and System A with the restricted domain alike and different?

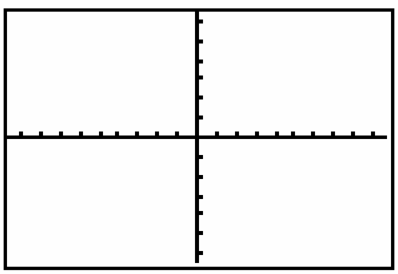
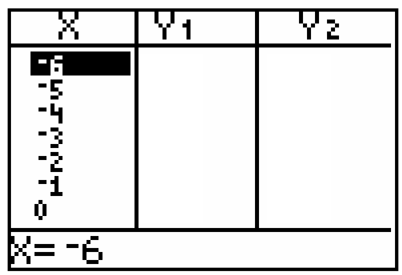
g) Why does System A without the restricted domain produce two solutions? Which solution of A is not a solution of the original system? What do we call that solution?

7. What understanding does graphing a system involving absolute value equations provide with regard to the actual number of solutions to the system and the corresponding equations that intersect? How does the graphical solution connect to the algebraic process?

8. Write a statement comparing the common algebraic process for solving absolute value equations to the conceptual understanding of the solutions to a system of absolute value equations.

9. Consider the system $|x+3| > 4$.

   a) Show the solution graphically, tabularly, and symbolically.

   b) When we ask students to show us where $|x+3| > 4$, what are we asking?